647510 **BSc4MatC401x** **Seat No :_____**

B.Sc. Semester - 4 (*CBCS*) Examination
March/April-2025 [NEW COURSE]
Mathematics-M401(Core)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que.1 (A) Answer any two out of three (10)

1. Discuss the convergence of sequence {Sn} Where Sn= $1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\ldots..+\frac{1}{n!}$.
2. If $\{a_n\}$ $\{b_n\}$ be two sequence such that $\lim_{n\to\infty} an$ =a, $\lim_{n\to\infty} bn$ =b then prove that $\lim_{n\to\infty} an\,.\,bn$ = a.b
3. Discuss the convergence of sequence {Sn} Where Sn= $1+\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\ldots..+\frac{1}{3^n}$.

Que.1 (B) Answer any one out of two (04)

1. Discuss the convergence of sequence $\{\sqrt{n+1}-\sqrt{n}\}$
2. Discuss the convergence of sequence $\{\frac{\sqrt{n}}{n+1}\}$

Que.2 (A) Answer any two out of three (10)

1. Discuss the convergence of series $1+2^2x+3^2x^2+4^2x^3+\ldots\ldots$
2. Discuss the convergence of series $1+\frac{3}{7}x+\frac{3.6}{7.10}x^2+\frac{3.6.9}{7.10.13}x^3+\ldots..$
3. Find the radius of convergence and interval of convergence of the series. $\sum_{n=0}^{\infty}\frac{(-3)^n x^n}{\sqrt{n+1}}$.

Que.2 (B) Answer any one out of two (04)

1. Show that the series $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots..+\frac{1}{2^{n-1}}+\ldots\ldots$ is convergent.
2. Test the convergence of $\frac{1}{1+2}+\frac{2}{1+2^2}+\frac{3}{1+2^3}+\ldots..$.

Que.3 (A) Answer any two out of three (10)

1. Find Linear Transformation T: $R^4\to R^3$ such that $T(e_1)=(1,1,1)$, $T(e_2)=(1,-1,1)$, $T(e_3)=(1,0,0)$, $T(e_4)=(1,0,1)$.
2. Prove that a Linear Transformation T: U→V is one-one if $N_T=\{\theta\}$.
3. For Linear Transformation T: $R^3\to R^3$, T(x,y,z) = (x-y+z, y-z, z) find T^{-1} if exist.

Que.3 (B) Answer any one out of two (04)

1. In usual notation prove that R_T is a subspace of V.
2. Find [T;B1,B2] for T: $R^2\to R^2$, T(x,y) = (x, -y) where B1= { e_1, e_2 } , B2={ (1,1), (1,-1)}.

Que.4 (A) Answer any two out of three (10)

1. Verify that $u.v=2u_1v_1+u_2v_2+4u_3v_3$ is inner product on R^3 or not.
2. Using Gram-schmidt process, transform the basis {(1,1,1), (-1,1,0), (1,2,1)} in to orthonormal basis.
3. State and prove Cauchy Schwartz's inequality.

Que.4 (B) Answer any one out of two (04)

1. Verify that $u.v=u_1^2v_1^2-u_2^2v_2^2+u_3^2v_3^2$ is inner product on R^3 or not.
2. State and prove Triangle inequality.

Que.5 (A) Answer any two out of three (10)

1. In usual notation derive radius of curvature ρ for Cartesian curves.
2. Find the radius of curvature ρ at any point of x=a(θ+Sinθ); y=a(1-Cosθ).
3. Find the radius of curvature ρ at (-2a, 2a), a>0 of $x^2y=a(x^2+y^2)$ using Newton's method.

Que.5 (B) Answer any one out of two (04)

1. Prove that $y=e^x$ is concave upwards everywhere and y=log x is convex upwards everywhere.
2. Find oblique asymptotes of y= $\frac{x^2+2x-1}{x}$.

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